2025

(Nov–Dec)

FYUP : 1st Semester Examination

MINOR

MATHEMATICS

(Fundamental Mathematics—I)

MTH–100

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Evaluate : 3+3=6

(i) $\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3}$

(ii) $\lim_{x \to \infty} \frac{2x^2 - x + 6}{3x^2 + 2x + 1}$

(b) Let

$$f(x) = x, \quad \text{for} \quad x = 0$$
$$= \tfrac{1}{2} - x, \quad \text{for} \quad 0 < x < \tfrac{1}{2}$$

Is the function continuous at $x = 0$? 3

(c) State Intermediate Value theorem. Use the theorem to show that $x^3 - x^2 + 2x - 1 = 0$ has a root in $(0, 1)$. 1+3=4

(d) Use ε-δ definition to show that

$$\lim_{x \to 2} (2x + 1) = 5$$ 5

2. (a) Evaluate : 3+3=6

(i) $\lim_{x \to \infty} \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

(ii) $\lim_{x \to 0} \dfrac{(1 + x)^2 - 1}{x}$

(b) Prove that the function defined by

$$f(x) = 0 \quad \text{for} \quad x^2 > 1$$
$$= \tfrac{1}{2} \quad \text{for} \quad x^2 = 1$$
$$= 1 \quad \text{for} \quad x^2 < 1$$

has discontinuities at $x = \pm 1$. 5

(c) Show that the function

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

has a removable discontinuity at $x = 1$. 3

(d) State Fixed Point theorem. Find the fixed point of the function

$$f(x) = \frac{1}{2} x + 1$$ 1+3=4

UNIT—II

3. (a) Find the derivatives of—

(i) x^x with respect to x;

(ii) x^6 with respect to x^3. 3+3=6

(b) Find the range of values of x for which the function

$$f(x) = x^3 - 6x^2 - 36x + 7$$

increases with x. 4

(c) Differentiate n times the equation

$$(1 + x^2) y_2 + y_1 = 0$$ 4

(d) Find the value of ξ in the mean value theorem

$$f'(\xi) = \frac{f(b) - f(a)}{b - a}$$

where $f(x) = x^2 + 3x + 2$, $a = 1$, $b = 2$ and $1 < \xi < 2$. 5

4. *(a)* State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = x^2$ in the interval $-1 \le x \le 1$. 2+4=6

(b) If $x^y = y^x$, find $\frac{dy}{dx}$. 3

(c) Show that $x > \sin x$ for $0 < x < \frac{\pi}{2}$. 5

(d) Find the nth derivative of $y = \sin x$. Hence find the 4th derivative of y at $x = 0$. 4+1=5

UNIT—III

5. *(a)* Evaluate : 5

$$\int_0^{\pi/2} (a^2 \cos^2 x + b^2 \sin^2 x)\, dx$$

(b) Deduce the reduction formula for $\int \cos^n x\, dx$, n being a positive integer greater than 1. 5

(c) If

$$f(x) = \begin{cases} \cos x, & -\frac{\pi}{2} \le x \le 0 \\ \sin x, & 0 < x \le \frac{\pi}{2} \end{cases}$$

show that $\int_{-\pi/2}^{\pi/2} f(x)\, dx = 2$. 4

(d) Find by integration, the area bounded by the parabola $y^2 = 4x$ and its latus rectum. 5

6. *(a)* Apply the definition of a definite integral as the limit of a sum to evaluate

$$\int_2^3 3x\, dx$$ 5

(b) Find the perimeter of the circle $x^2 + y^2 = a^2$ using integration. 5

(c) Evaluate : 3

$$\int_{-1}^{1} (x)\, dx$$

(d) Obtain the reduction formula for $\int x^n e^{ax} dx$, n being a positive integer. Hence find $\int x^2 e^{ax} dx$. 3+3=6

UNIT—IV

7. (a) If z_1 and z_2 are two complex numbers, then show that $|z_1 z_2| = |z_1||z_2|$ and $\text{amp}(z_1 z_2) = \text{amp} z_1 + \text{amp} z_2$. 5

(b) Find the values of $(1-i)^{1/3}$. 5

(c) Apply synthetic division to find the quotient polynomial and the remainder when $3x^5 - 4x^4 + 6x^3 - 9x + 2$ is divided by $x+3$. 4

(d) If α, β, γ are the roots of the equation $x^3 + px + q = 0$, find the value of

$$\sum \frac{1}{\alpha+\beta}$$ 5

8. (a) Solve the cubic equation $x^3 - 6x - 9 = 0$ by Cardan's method. 6

(b) Express $x^5 + 5x^3 + 3x$ as a polynomial in $(x-1)$. 4

(c) Solve the equation

$$x^3 - 15x^2 + 6x - 80 = 0$$

given that the roots are in arithmetic progression. 5

(d) Prove that the equation $x^6 - 3x^2 - x + 1 = 0$ has at least two imaginary roots. 4

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